Abstract

The partitioning of a tandem layout for an automated guided vehicle (AGV) system directly affects the system performance. Determining the location of the transfer points between zones is another important issue in the design of a tandem layout. In this paper, a heuristic partitioning algorithm for a tandem AGV system based on the concept of variable path routing within a zone is developed. Transfer points are simultaneously determined during the network partitioning process. The algorithm assumes the prior existence of a conventional and unidirectional layout. The problem of interest is one of converting the existing, conventional layout to a variable path based tandem AGV system layout. A comparison between the conventional and the tandem AGV system with respect to system performance under the same operating conditions is presented. The results show that the variable path based tandem layout requires a smaller number of vehicles in most situations than the equivalent conventional layout. In addition, the total vehicle use time of a tandem layout is smaller than that of the conventional layout.

Keywords: Automated Guided Vehicles, Neighboring, Partitioning, Tandem, Variable Path

Introduction

The use of flexible manufacturing systems (FMSs) has placed a greater demand on automated material handling systems. Among the various material handling devices available, automated guided vehicles (AGVs) have become an essential part of FMSs because of their flexibility and adaptability for material handling.

Automated guided vehicle system layouts can be classified into two main types, namely, conventional layout and tandem layout. In a conventional layout, several vehicles are used and each vehicle is allowed to serve any station in the system. Implementation of such a system requires consideration of several important issues, including flow path design, number of vehicles required, location of pickup and drop-off points, vehicle dispatching, and traffic control strategy.

Tandem AGV layout was first proposed by Bozer and Srinivasan (1989, 1991). The tandem configuration is an application of the “divide-and-conquer” principle to AGV systems. It is based on the partitioning of all stations into nonoverlapping, single-vehicle closed loops or zones with additional transfer points provided as interface between adjacent loops. All stations within the same loop are served by only one dedicated vehicle. Each transfer point allows loads to be moved bidirectionally between adjacent loops.

The advantages of tandem AGV systems are the ease of control and reduction in system complexity. No traffic congestion or conflict occurs in a tandem AGV system. Vehicles can travel bidirectionally within an aisle because only one vehicle operates in a loop. The disadvantages are increased number of load pickups and drop-offs, increased floor space requirement, increased transfer station installation cost, and decreased tolerance for system breakdowns.

Although tandem AGV systems are competitive with conventional AGV systems from a throughput standpoint (Bozer and Srinivasan 1992), it is evident that the results obtained significantly depend on the partitioning method and the production routing of the jobs through the tandem layout. Determining the location of the transfer points is an important issue in the design of a tandem layout. A partitioning algorithm can enable the grouping of workstations into single-vehicle regions while minimizing the number of regions and satisfying the required workload.

In this paper, the use of variable paths instead of loops in tandem AGV layouts is examined. In the existing literatures, tandem layouts are generally associated with loop layouts. There is no reported work on tandem layout that has explicitly considered or analyzed the effect of using variable path routing within a zone similar to that employed in a general network. Similar to a loop in a tandem system, a
variable path tandem zone also has a single dedicated vehicle. A variable path tandem zone is also composed of multiple stations; however, vehicle movement between points in a variable path zone is not necessarily along a loop, but along the shortest path within the zone. Because each zone is dedicated to one vehicle, its traffic control is far more simplified than that of a conventional layout. Furthermore, because a variable path zone is a mini-network similar in characteristics to a conventional layout, it offers vehicle routing flexibility not present in a tandem loop layout. Transfer points are used to move loads in and out of a zone similar to a tandem loop system. With variable path zone layout, pickup and drop-off points do not all have to be arranged on a loop; pickup and drop-off points can be in the interior of a zone.

A heuristic partitioning algorithm for a tandem AGV system based on the concept of variable path zones is developed in this paper. Transfer points are simultaneously decided on during the network partitioning process. The algorithm assumes the prior existence of a conventional and unidirectional layout or a model of such layout. The problem of interest in this paper is one of converting an existing, conventional layout or a model of such layout to a variable path tandem AGV system network. The primary design criterion employed is the minimization of the number of variable path tandem AGV system zones or vehicles for a given conventional layout subject to satisfying the transport demands within a specified time interval. Therefore, it is possible to make a direct comparison between the conventional and variable path tandem layout with respect to system performance under the same operating conditions.

Minimizing the number of zones does not necessarily ensure that the transport workload will be balanced among the zones. In support of the objective of minimizing the number of zones while at the same time achieving workload balance between the zones, a procedure to design a system that achieves near-workload balance between zones is also presented. The workload balance is obtained by having the number of zones fixed at the minimum level obtained in the initial solution.

**Literature Review**

The implementation of a conventional AGV system requires the consideration of several important issues, including flow path design, number of vehicles required, location of pickup and drop-off points, vehicle dispatching, and traffic control strategy.

The flow path design of AGV systems establishes the location and direction of vehicle travel. Gaskins and Tanchoco (1987), Goetz and Egbelu (1990), Kaspi and Tanchoco (1990), Sinriech and Tanchoco (1991), Venkataramanan and Wilson (1991), and Seo and Egbelu (1995) discussed the flow path design issue.

The number of vehicles required is a function of the total loaded and empty vehicle travel time, blocking time, congestion time, breakdown rates, and pickup and drop-off time. This issue was presented in the literature by Maxwell and Muckstadt (1982); Newton (1985); Egbelu (1987); Tanchoco, Egbelu, and Taghaboni (1987); Leung, Khator, and Kimbler (1987); Mahadevan and Narendran (1990); and Lin (1990).

The location of pickup and drop-off points is a very important design issue in an AGV system because it is directly related to the total loaded and empty travel distances of AGVs. Actually, the sources and destinations of vehicles are generally from the pickup points to the drop-off points of the departments. Montreuil and Ratliff (1988), Kiran and Tansel (1989), Goetz and Egbelu (1990), Sinriech and Tanchoco (1992), and Kim and Klein (1996) addressed how to locate pickup and drop-off points.

Vehicle dispatching is generally governed by a set of rules that determine which vehicle to use for a particular operation. Egbelu and Tanchoco (1984); Hodgson et al. (1987); Bartholdi and Platzman (1989); Malmborg (1990); Zeng, Wang, and Jin (1991); and Kim and Tanchoco (1991) dealt with the problem of vehicle dispatching and traffic control.

Bozer and Srinivasan (1989) proposed tandem AGV systems. Tandem AGV systems are studied by Bozer and Srinivasan (1991, 1992); Lin, Chang, and Liu (1994); Lin and Dgen (1994); Hsieh and Sha (1997); Huang (1997); and Liu and Chen (1997).

Although a tandem AGV layout is very competitive with a conventional AGV layout (Bozer and Srinivasan 1992), research focusing on the tandem layout is less compared to that of the conventional layout. There are several important issues to consider when designing a tandem layout. One of the important issues is the development of a partitioning algorithm that enables an analyst to group worksta-
tions into single vehicle zones subject to minimizing the number of zones and satisfying the transport demand or workload. Another design issue is that of determining the location of the transfer points. These two design issues are very important because they affect the system performance directly. Currently, there is less reported research on the partitioning of AGV networks for a tandem layout and the location of the transfer points. In most previous research studies, the models assume that the tandem layout and the transfer points are given. There has also been a limited availability of tools for the design of tandem AGV systems. The lack of a satisfactory method of resolution of the problems partly motivated the work undertaken in this paper.

Model Development

In a conventional layout, there are multiple workcenters in the system. Each workcenter has a pickup point and a drop-off point, which, respectively, serve as a point of entry for incoming jobs and an exit point for finished parts. The pickup and drop-off points need not be distinct but can coincide in location. A variable path tandem layout is based on partitioning all the stations into non-overlapping, single-vehicle closed subnetworks with additional transfer points provided as interface between adjacent subnetworks. A subnetwork or zone is assigned a dedicated vehicle. A subnetwork is a subset of a parent conventional network layout and has similar connectivity characteristics as its parent network. Therefore, a subnetwork is simply a distinct section of a larger network that communicates with other parts of the main network only through transfer points. Some network modification may be necessary to convert a conventional layout to a variable path tandem layout.

In this paper, a procedure to partition a conventional layout into variable path tandem layout or zones (subnetworks) is presented. Movement of a vehicle within a variable path zone is along the shortest path within the subnet. Because each variable path zone is dedicated to one vehicle, traffic control is simpler than that of a conventional layout. Furthermore, because travel is along the shortest path, total material handling time is shorter than that of a tandem loop layout.

The following definitions are used in developing a variable path zone or tandem layout.

Zone—a station or a set of stations that are contiguous and whose total workloads can be handled by one and only one dedicated vehicle. There are two types of zones, namely, complete zones and formative zones.

Complete Zone—a zone whose size is already fixed in the sense that all stations in the zone are known. No new station can be added to the complete zone because the workload of the zone will then exceed the capacity of one vehicle if any new station is added.

Formative Zone—a zone whose size is not completely defined and therefore is subject to change through the addition of new stations. A formative zone can therefore be expanded.

Dummy Station—a block of the plant floor without a station. A machine in a dummy station is assigned a dummy identification number. A dummy station is needed for the expansion of a formative zone.

Starting Station—a station that is the first to be selected when a new zone is to be formed.

Expanded Station—a station that is a candidate for addition to a formative zone.

Neighboring Station—an adjacent station or a station that shares a common boundary with a target station. If a station is already included in a complete zone, it cannot be a neighboring station of an unassigned station.

Unassigned Station—a station that is not yet assigned to any zone.

Isolated Station—all unassigned stations have to form a contiguous region. If there is a station or a set of stations that are not contiguous to the other unassigned stations, they are referred to as isolated stations. The following method can be used to find an isolated station: start from one of the unassigned stations and place it in a set. Find the neighboring stations of the elements (stations) in the set until all neighboring stations of all elements are included in the set. If an unassigned station remains that is not included in the set, such a station is an isolated station.

The following assumptions are made in this research.

1. The number of jobs and the operation sequence of each job are known.
2. An AGV moves a unit load, and once the system is in progress, a unit load cannot be broken down into smaller unit loads or combined with other unit loads to form a larger unit load in the system.
3. An AGV moves only one unit load at a time.
4. The movement of an AGV between two points in a zone is along the shortest path.
5. The ratio of empty and loaded vehicle speeds is constant.
6. The buffer sizes at pickup, drop-off, and transfer points are infinite.
7. The aisles are wide enough for placing multiple parallel guidepaths. This allows for parallel guidepaths to be laid at the boundary of two zones.
8. The intersection of aisles has sufficient space for placing a transfer point.
9. Each zone has at least one transfer point to communicate with other zones.
10. At most, only one new transfer point can be installed when a new zone is formed.
11. There is a single input/output (pickup/drop-off) point for each station.

To create a variable path tandem layout, a partitioning algorithm was developed and applied to an existing unidirectional conventional layout or a model of such layout using job flow information. The proposed partitioning algorithm generates zones sequentially. The algorithm starts from one of the stations in the system to form the first zone. It then defines a transfer point for the first zone and calculates its workload. New stations are added to the zone sequentially until a further addition would exceed the handling capacity of one vehicle. As a zone expands, the location of the transfer point is reevaluated to determine the best point at which to position it. When a zone can no longer be expanded, a new zone is initiated from an unassigned station and the process is repeated. The creation and expansion of new zones is continued until all stations are assigned to zones. Thereafter, the algorithm terminates with a final solution.

In the assignment of stations to zones, it is possible that the workload distribution among the zones or vehicles is unbalanced in the final solution obtained by the process above. This paper also presents a procedure to improve on the final solution from the process above to obtain a more balanced workload among the zones while keeping the number of zones fixed at the level of the final solution.

In the design of a variable path zone or tandem layout, four important questions arise:

1. How does one choose a starting station?
2. How does one choose a transfer point?
3. How does one calculate the workload of each zone?
4. How does one choose a station to which a zone can expand?

Each of these issues is addressed further below.

**Choice of Starting Station**

To start the construction of a zone, the heuristic algorithm must select a starting workstation to initiate the zone. The need for selecting an initial workstation occurs in one of two cases. The first case (i.e., case 1) occurs at the initiation of the algorithm to form the first zone. In the second case (i.e., case 2), there is already a complete zone, and therefore at least one transfer point already exists in the system.

**Case 1**

In case 1, there is no complete zone in the system. This case occurs only one time and at the very first stage during the execution of the partitioning algorithm. The partitioning algorithm starts by choosing a border station that has the minimum number of neighboring stations. If there is a tie, the station with the maximum flow is chosen. This flow is defined as the sum of in-flow and out-flow. This choice of the starting station as described decreases the possibility of creating an isolated station.

**Case 2**

In case 2, there is already a complete zone in the system, and at least one transfer point exists. In this case, one of the unassigned stations whose boundary includes the transfer point is chosen as a starting station. Of all stations whose boundaries include a transfer point, choose a station that has the smallest number of neighboring stations. If there is a tie, choose the station that has the largest flow rate between that station and the transfer point. The flow rate of a transfer point implies the rate of flow that has to move through that transfer point.
Choice of Transfer Point

In this paper, a transfer point can only be located at the intersection of aisles. For each intersection, count the number of unassigned neighboring stations whose boundaries include that intersection. Choose the intersection that has the largest number of unassigned neighboring stations as a transfer point. If there is a tie, calculate the flow rate of those tied intersections. The flow rate of an intersection is the sum of all flows through that intersection to its neighboring stations. A neighboring station means an adjacent station or a station that shares a common boundary. Choose the intersection that has the largest flow rate as a transfer point.

If a zone is enlarged, and therefore the zone size is changed, all candidate transfer points must be reevaluated to determine the new best point at which to locate a transfer point. Only one transfer point is required for a new zone. This implies that as a new point qualifies as the best location for a transfer point, the previous transfer point for the zone is eliminated.

Vehicle Workload

Egbelu (1987) described an analytical approach for estimating the number of vehicles required for an AGV system. In this research, the vehicle workload for ‘zone i’ is calculated as in Eqs. (1) to (5), based on Egbelu’s fourth approach. Maximum capacity of a vehicle is calculated as in Eq. (6).

\[
D'_{jk} = f_{jk} d_{jk}
\]

\[
g_{jk} = \left( \sum_{p=1}^{n} f_{pj} \right) \left( \sum_{p=1}^{n} f_{kp} \right) / \left( \sum_{p=1}^{n} \sum_{m=1}^{n} f_{pm} \right)
\]

\[
D''_{jk} = g_{jk} d_{jk}
\]

\[
D_{jk} = D'_{jk} + D''_{jk}
\]

\[
W_i = \frac{\sum_{j=1}^{n} \sum_{k=1}^{n} D_{jk}}{v} + \left( \sum_{j=1}^{n} \sum_{k=1}^{n} f_{jk} \right) (t_p + t_d)
\]

where

\[D'_{jk} = \text{total distance of loaded runs from pickup point } j \text{ to drop-off point } k\]
\[f_{jk} = \text{flow rate from pickup point } j \text{ to drop-off point } k \text{ where both pickup point } j \text{ and drop-off point } k \text{ are within } \text{zone } i\]
\[d_{jk} = \text{travel distance for flow } f_{jk} \text{ within } \text{zone } i\]
\[g_{jk} = \text{number of empty runs from drop-off point } j \text{ to pickup point } k\]
\[n_i = \text{number of pickup or drop-off points in } \text{zone } i' \text{ (Pickup or drop-off points in } \text{zone } i' \text{ include all workstations in } \text{zone } i' \text{ and all transfer points that are on the boundary of } \text{zone } i')\]
\[D''_{jk} = \text{total distance of empty runs from drop-off point } j \text{ to pickup point } k\]
\[D_{jk} = \text{total distance traveled between pickup point } j \text{ and drop-off point } k\]
\[W_i = \text{vehicle workload for } \text{zone } i \text{ in time units (e.g., 60 min.)}\]
\[v = \text{vehicle travel speed; uniform speed is assumed}\]
\[t_p = \text{pickup time per unit load}\]
\[t_d = \text{drop-off time per unit load}\]
\[C = \text{maximum capacity of a vehicle in time units}\]
\[T = \text{time unit (e.g., 60 min.)}\]
\[u = \text{maximum allowable vehicle utilization}\]

The reasoning behind this approach is that in a job shop environment the sequence at which load pickup requests are generated from the workstations is very random. If the sequence of order generation is random, it is argued that the sequence in which requests are satisfied will also tend to be random in the long run. Because vehicles are always freed from assignments at drop-off points, it is at these points that vehicle reassignment decisions are made. The next pickup point a vehicle is reassigned to is, therefore, a function of vehicle dispatching rules in force. With a fair dispatching rule, the above approach to calculating the vehicle workload is justified. There are other methods for estimating \(W_i\) that exist in literature (Egbelu 1987). A user can employ an alternate method for calculating \(W_i\) that the user considers most appropriate for his or her application or design philosophy. For example, the application of Egbelu’s second approach for calculating the vehicle workload is found in Yu (1997).

The workload generated in zone \(i\) can be handled by a vehicle if and only if \(W_i \leq C\). \(C\) is an estimate
of the maximum effective time available to a vehicle over a period. The definition of \( C \) assumes that all other sources of time losses (e.g., vehicle efficiency, delays, breakdowns, etc.) have been factored in.

The total loaded distance includes the loaded flow distance within a zone, the in-flow distance to a zone, out-flow distance from a zone, and transit flow distance through a zone. Transit flow is defined as a flow that passes through a zone but originates from another zone and terminates at another zone. The job being transported does not require processing in the transit zone during that leg of the trip. This occurs because of the tandem layout characteristics.

**Choice of Expanded Station**

To expand a zone, consider all stations that are neighbors to any station already included in the formative zone. For each station considered, calculate the flow rate between the considered station and the stations already included in the formative zone. Choose the station that has the largest flow rate as the new addition to the formative zone, provided that the workload of the new expanded zone does not exceed the handling capacity of a vehicle. If the workload of the expanded zone does exceed the capacity of a vehicle, choose the qualifying station with the maximum flow rate whose addition to the formative zone would not cause the workload of the zone to exceed the capacity of one vehicle. The selection criterion is based on the principle that if there is a large flow between a station and a formative zone, one would like to add this station to a formative zone instead of the other stations in order to decrease transit flow. If the flow rates are tied, choose the station that has the smallest number of unassigned neighboring stations. In a case in which the largest flow rates are the same for several stations, one may want to expand to a station that is located at the boundary of the system to prevent the occurrence of an isolated station. If the number of unassigned neighboring stations is also tied, choose a station arbitrarily.

**Heuristic Partitioning Algorithm for a Variable Path Tandem Layout**

The partitioning algorithm generates zones sequentially and dynamically. This algorithm starts from one of the stations in the system to initiate the first zone. This is followed by the creation of a transfer point for the first zone and the calculation of the workload for the zone. New stations are added to a zone until its handling workload can no longer be increased without exceeding the capacity of one vehicle. A complete zone is formed when no new neighboring station can be added to the zone without exceeding the capacity of one vehicle. If there exist stations not yet assigned to a zone, a new zone is started and the process of adding new stations to the formative zone is repeated. The algorithm ends when all workstations have been assigned to zones.

Based on the above characteristics, the partitioning algorithm is developed as follows using the following notations.

- \( i \): zone number
- \( j \): station number
- \( k \): intersection number
- \( Z_i \): zone \( i \). A zone contains a set of stations.
- \( S_j \): station \( j \)
- \( I_k \): intersection \( k \)
- \( O_i \): an ordered set of unassigned stations from which the starting station is to be selected
- \( Q_i \): an ordered set of unassigned neighboring stations to zone \( i \). The elements of \( Q_i \) are arranged in a non-increasing order of their flow rate with zone \( Z_i \).
- \( S_z \): a station whose traffic workload exceeds the capacity of one vehicle. Such station forms a self-contained zone. A local material handling device, such as a conveyor, can be used instead of an AGV to move items in and out of the station through the transfer points.
- \( Z_i' \): an exploratory expansion of \( Z_i \) when a new station is being evaluated for inclusion in zone \( Z_i \).

The following steps present the heuristic algorithm for generating a variable path tandem layout from a conventional layout.

**PARTITIONING ALGORITHM**

**STEP 1**

Let \( i = 1 \). \( Z_i = \{ \} \).

**STEP 2**

(a) Count each station’s number of neighboring stations.
(b) Place all stations in $O_s$ and arrange them in a nondecreasing order of the number of neighboring stations each station has.

(c) If a tie exists between stations, arrange the stations that are tied according to a nonincreasing order of the flow rate of those stations. If flow rate is also tied, arrange the tied stations arbitrarily.

$$O_s = \{O_s(1), O_s(2), \ldots\}$$

**STEP 3**

(a) If there is a station in $O_s$, go to step 5.

(b) Otherwise, initiate a new zone $Z_i$ as follows: of all unassigned stations whose boundaries include a transfer point, choose the station that has the smallest number of neighboring stations. If a tie exists, choose the station that has the largest flow rate between that station and the transfer point. Let this chosen station be $S_z$.

$$Z_i = \{S_z\}$$

For each intersection $k$ that is located on the boundary of zone $Z_i$, calculate the number of unassigned stations whose boundaries also include intersection $k$. Choose the intersection that has the largest number of unassigned neighboring stations as a transfer point. If a tie exists, calculate the flow rate of those tied intersections. This flow rate is the sum of all flow rates that move through the intersection between the zone and its unassigned neighboring stations. Choose the intersection that has the largest flow rate as a transfer point. If the flow rate is also tied, choose an intersection arbitrarily.

**STEP 4**

$Z_i$ is a self-contained zone whose workload exceeds one AGV. A different kind of material handling device, such as a conveyor, can be used for this zone to handle all flows associated with the station. The current transfer point location for the zone becomes its adopted official transfer point. Save the information on the in-flow, out-flow, and transit flow that move through the transfer point. Go to step 16.

**STEP 5**

Set $Z_i' = Z_i$. Remove the first station $O_s(1)$ from $O_s$ and place it in zone $Z_i'$.

$$Z_i' = \{O_s(1)\}$$

**STEP 6**

If there is no unassigned station remaining in the system, go to step 8. Otherwise, for each intersection $k$ that is located on the boundary of zone $Z_i'$, calculate the number of unassigned stations whose boundaries also include intersection $k$.

**STEP 7**

Choose the intersection that has the largest number of unassigned neighboring stations as a transfer point. If a tie exists, calculate the flow rate of those tied intersections. This flow rate is the sum of all flow rates that move through the intersection between the zone and its unassigned neighboring stations. Choose the intersection that has the largest flow rate as a transfer point. If the flow rate is also tied, choose an intersection arbitrarily.

**STEP 8**

Calculate the vehicle workload for zone $Z_i'$ as defined in Eqs. (1) to (5).

**STEP 9**

If the vehicle workload for zone $Z_i'$ is less than the maximum capacity of a vehicle as defined in Eq. (6), set $Z_i = Z_i'$. Otherwise go to step 12.

**STEP 10**

If there is no unassigned station in the system, stop. You have the solution for the variable path tandem layout system. Otherwise continue in step 11.

**STEP 11**

(a) Place all unassigned stations that share boundaries with zone $Z_i'$ in $Q_i$. Arrange the stations in a non-increasing order of the flow rate between zone $Z_i'$ and each of the stations in $Q_i$, respectively.
(b) If a tie exists, arrange the tied stations in nondecreasing order according to the number of unassigned neighboring stations.

(c) If a tie still exists, arrange the tied stations in nondecreasing order of the flow rates of tied stations.

(d) If a tie still exists, break the tie arbitrarily.

Go to step 14.

STEP 12

Set $Z_i' = Z_i$. If zone $Z_i'$ is null, \{\}, go to step 3. Otherwise, continue in step 13.

STEP 13

If there is no station in $Q_i$, go to step 15. Otherwise, continue in step 14.

STEP 14

Remove the first station $Q_i(1)$ from $Q_i$ and place it in the zone $Z_i'$. If isolated stations emerge during the execution of this step, place the isolated stations also in zone $Z_i'$.

$$Z_i' = \{ Z_i, Q_i(1) \}$$

Go to step 6.

STEP 15

$Z_i$ is a complete zone. Only one vehicle will operate within this zone. The current transfer point location for the zone becomes its adopted official transfer point. Save the information on the in-flow, out-flow, and transit flow that move through the transfer point.

STEP 16

Set $i = i + 1$. $Z_i = \{ \}$. 

STEP 17

Identify the unassigned stations whose boundaries include a transfer point. Count the number of neighboring stations to these stations and arrange the stations in $O_s$ in a nondecreasing order according to the number of neighboring stations. If a tie exists, calculate the flow rate between the tied stations and the transfer point and arrange the tied stations in a non-increasing order of the flow rate.

$$O_s = \{ O_s(1), O_s(2), \ldots \}$$

Go to step 3.

Figure 1 describes the algorithmic steps of the design of a variable path tandem layout.

Examples and Results

To illustrate the application of the algorithm, two examples are presented. The first example involves a facility with 10 stations and four jobs. The job routings are randomly generated. The second example is drawn from Bozer (1992) and involves 20 stations and six jobs. The performances of the resulting variable path tandem layouts are compared with their conventional layout counterparts. Analysis of the results is presented. Finally, the problem of balancing the vehicle utilization among the zones is also presented.

The comparison of the two layout types is based on the number of vehicles required to satisfy the move demands in a facility within a fixed time interval while simultaneously maintaining workload feasibility of a vehicle. Such interval can be a day, a shift, or multiple days or shifts. Comparison based on fleet size requirements is chosen because it is the natural equivalence of the number of tandem zones required in a tandem layout. Other criteria can be used for comparison as well, provided they are consistent with the design objective.

Example 1

Variable Path Tandem Layout for Example 1

The layout shown in Figure 2 is considered here. The job sequences and the production rate of each job are presented in Table 1. Table 2 shows the $(x, y)$ coordinates of the stations on the shop floor, where the origin of the network coordinate frame is at the lower left corner. The coordinates of the intersect-
Figure 1
Graphical Representation of Design Procedure for a Variable Path Tandem Layout
tions are shown in Table 3. Assume that the loaded or empty vehicle speed is 60 m/min., the pickup or drop-off time for each load is 0.25 min., and the maximum capacity per vehicle in the system is 60 min./hr. This implies that 100% utilization per vehicle is permitted. A utilization level of 100% implies that maintenance takes place during off hours and that no vehicle breakdowns occur.

The shortest distance between two stations or a station and an intersection can be derived from Table 2, Table 3, and Figure 2. A From-To chart of job flows can be derived from Table 1. Neighboring stations can be identified based on the definition of neighboring stations as provided earlier. For example, the neighboring stations for station 4 are stations 3 and 6.

Using the shop floor information provided, the network partitioning algorithm was applied. Figure 3 shows the corresponding tandem layout obtained by the partitioning algorithm. Zone 1 is sequentially formed by stations 1 and 7. Stations 5 and 2 are sequentially added to zone 2. Zone 3 is sequentially formed by stations 8, 9, 10, and 6. Zone 4 is formed by stations 4 and 3. Intersections 7, 10, and 12 are used as transfer points. The total distance of loaded runs, the total distance of empty runs, total operation time of AGV, and the vehicle utilization in the zones are presented in Table 4.

Comparison of Variable Path Tandem Layout with Conventional Layout for Example 1

As seen from the analysis, the variable path tandem layout requires four zones. Therefore, four vehicles are required to operate the system. The average utilization per vehicle is equal to (95.78% + 91.58% + 90.75% + 62.22%) / 4 = 85.08%. The use of average vehicle utilization as a measure is meaningful only within the design context presented in this paper, which requires that the workload within a zone must be satisfied by only one vehicle and that the workload cannot exceed 100% or C.

The number of vehicles required for a conventional layout can be calculated according to Eqs. (1) through (5), but modified slightly as shown in Eqs. (7) through (12).

\[ D_{jk} = f'_{jk} d'_{jk} \]
\[ g_{jk} = \left( \sum_{p=1}^{n} f^*_{jp} \right) \left( \sum_{p=1}^{n} f^*_{pm} \right) \]
\[ D'_{jk} = g'_{jk} d'_{jk} \]

<table>
<thead>
<tr>
<th>Job Type</th>
<th>Process Route</th>
<th>Production Rate/Hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 - 3 - 6 - 10 - 9 - 2</td>
<td>3.0</td>
</tr>
<tr>
<td>B</td>
<td>1 - 7 - 10 - 8 - 1</td>
<td>3.0</td>
</tr>
<tr>
<td>C</td>
<td>1 - 5 - 6 - 7 - 3 - 2</td>
<td>3.0</td>
</tr>
<tr>
<td>D</td>
<td>1 - 9 - 4 - 7 - 1</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 1

Job Sequences and Production Rate for Example 1

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate</td>
<td>(8, 75)</td>
<td>(40, 65)</td>
<td>(70, 75)</td>
<td>(100, 70)</td>
<td>(60, 35)</td>
<td>(85, 25)</td>
<td>(1, 15)</td>
<td>(45, 1)</td>
<td>(60, 15)</td>
<td>(95, 1)</td>
</tr>
</tbody>
</table>

Table 2

Coordinates of the Stations for Example 1

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate</td>
<td>(60, 75)</td>
<td>(80, 75)</td>
<td>(1, 50)</td>
<td>(20, 50)</td>
<td>(40, 50)</td>
<td>(60, 50)</td>
<td>(80, 50)</td>
<td>(100, 50)</td>
<td>(1, 25)</td>
<td>(20, 25)</td>
</tr>
</tbody>
</table>

Table 3

Coordinates of the Intersections for Example 1

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate</td>
<td>(60, 75)</td>
<td>(80, 75)</td>
<td>(1, 50)</td>
<td>(20, 50)</td>
<td>(40, 50)</td>
<td>(60, 50)</td>
<td>(80, 50)</td>
<td>(100, 50)</td>
<td>(1, 25)</td>
<td>(20, 25)</td>
<td>(40, 25)</td>
<td>(60, 25)</td>
<td>(80, 25)</td>
<td>(100, 25)</td>
<td>(60, 1)</td>
<td>(80, 1)</td>
</tr>
</tbody>
</table>
Based on the total transport workload of the system, the number of vehicles required is given by $N$, where

$$N = \left\lceil \frac{W}{Tu} \right\rceil$$

(12)

where

- $N =$ number of vehicles required
- $T =$ time unit (e.g., 60 min.)
- $u =$ maximum allowable vehicle utilization
- $\lceil x \rceil =$ smallest integer greater than or equal to $x$

Based on the above set of equations and the parameter values used for the tandem system, the total distances of loaded runs and empty runs for the conventional layout are 7182.00 m and 6397.00 m, respectively. The total number of pickup and drop-off is 54, respectively. Therefore, the number of vehicles required for a conventional layout is equal to 5, as calculated below.

$$W = \frac{7182.00 + 6397.00}{60} + 54(0.25 + 0.25) = 253.32 \text{ min.}$$

$$N = \left\lceil \frac{253.32}{(60)(1.0)} \right\rceil = \lceil 4.22 \rceil = 5 \text{ vehicles}$$

Thus, the vehicle requirement under the conventional layout is 5. The average vehicle utilization, $\mu$, equals 84.44%, where

$$\mu = \frac{W}{TN} = \frac{253.32}{(60)(5)} = 84.44\%$$

The above calculation does not consider time lost that may result from vehicle blocking usually encountered in multivehicle conventional layouts. If

### Table 4

<table>
<thead>
<tr>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stations</td>
<td>1, 7</td>
<td>2, 5</td>
<td>6, 8, 9, 10</td>
</tr>
<tr>
<td>Transfer Points</td>
<td>10</td>
<td>10, 12</td>
<td>7, 12</td>
</tr>
<tr>
<td>Loaded Distance</td>
<td>1494.00</td>
<td>1380.00</td>
<td>1188.00</td>
</tr>
<tr>
<td>Empty Distance</td>
<td>1054.20</td>
<td>927.27</td>
<td>1088.73</td>
</tr>
<tr>
<td>Number of Pickup</td>
<td>30.0</td>
<td>33.0</td>
<td>33.0</td>
</tr>
<tr>
<td>Number of Drop-off</td>
<td>30.0</td>
<td>33.0</td>
<td>33.0</td>
</tr>
<tr>
<td>Total Operation Time of AGV</td>
<td>57.47 min.</td>
<td>54.95 min.</td>
<td>54.45 min.</td>
</tr>
<tr>
<td>Vehicle Utilization</td>
<td>95.78%</td>
<td>91.58%</td>
<td>90.75%</td>
</tr>
</tbody>
</table>
blocking time can be properly accounted for, the number of vehicles required for a conventional layout could increase while the number of vehicles required for a tandem layout stays the same. No blocking occurs in tandem layouts because the vehicles are assigned dedicated zones.

Therefore, based on the results above, it can be concluded that the tandem layout is superior to the conventional layout for the case presented. This result also suggests that opportunity exists to redesign or repartition the tandem layout so as to balance the workload between the zones or vehicles. This kind of redesign is necessary if, for example, a secondary design objective is pursued as well. Such secondary objective can be the balancing of workloads among the zones and, consequently, the utilization of vehicles. Balancing the workload between vehicles will improve material flow and reduce mean job flow time, mean time in queue, mean queue length, and parts outages at workstations. At high vehicle utilization level, flow delays can be expected as well as increased output queue lengths. Similarly, unbalanced workload can result in an increase in production makespan caused by congestion in one or more tandem zones.

**Example 2**

*Variable Path Tandem Layout for Example 2*

The layout of example 2, shown in Figure 4, was used by Bozer and Srinivasan (1992) in their paper on the tandem AGV system. Job sequences and production rates of each job are presented in Table 5. Table 6 shows the \( (x, y) \) coordinates of the stations on the floor, where the origin of the network coordinate frame is located at the upper left corner. The coordinates of the intersections are shown in Table 7. Note that intersections 1, 3, 11, 14, and 17 are not intersections in the true sense, but are considered intersections because there is no intersection between stations 1 and 4, stations 2 and 3, stations 11 and 12, stations 14 and 18, and stations 17 and 20, respectively.

Assume that the loaded or empty vehicle speed is 15 distance units/min., and the pickup or drop-off time for each load is 0.2 min. The maximum capacity per vehicle in the system is 60 min./hr.

After applying the partitioning algorithm, the tandem layout shown in Figure 5 is obtained. Stations 1, 4, 5, 2, 3, and 6 are sequentially added to zone 1. Zone 2 is sequentially formed by stations 10, 14, 18, 15, 11, and 12. Stations 19, 16, 20, 17, 13, and 8 are sequentially added to zone 3. Zone 4 is sequentially

---

**Table 5**

<table>
<thead>
<tr>
<th>Job Type</th>
<th>Job Route</th>
<th>Job/HR</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 - 3 - 6 - 2 - 5 - 4 - 1</td>
<td>1.5</td>
</tr>
<tr>
<td>B</td>
<td>1 - 6 - 8 - 7 - 9</td>
<td>1.5</td>
</tr>
<tr>
<td>C</td>
<td>9 - 7 - 8 - 16 - 20 - 17 - 13 - 9</td>
<td>1.5</td>
</tr>
<tr>
<td>D</td>
<td>18 - 15 - 11 - 12 - 16</td>
<td>1.5</td>
</tr>
<tr>
<td>E</td>
<td>18 - 15 - 19 - 12 - 11</td>
<td>1.5</td>
</tr>
<tr>
<td>F</td>
<td>18 - 14 - 10 - 4 - 5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Table 6**

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate ( (7, 1) )</td>
<td>(22, 1)</td>
<td>(30, 1)</td>
<td>(1, 5)</td>
<td>(7, 9)</td>
<td>(25, 9)</td>
<td>(45, 9)</td>
<td>(37, 15)</td>
<td>(55, 15)</td>
<td>(7, 22)</td>
<td></td>
</tr>
<tr>
<td>Coordinate ( (22, 22) )</td>
<td>(30, 22)</td>
<td>(45, 22)</td>
<td>(1, 30)</td>
<td>(16, 30)</td>
<td>(37, 30)</td>
<td>(55, 30)</td>
<td>(7, 37)</td>
<td>(25, 37)</td>
<td>(45, 37)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7**

| Coordinate \( (1, 1) \) (16, 1) (25, 1) (37, 1) (1, 9) (16, 9) (37, 9) (55, 9) (1, 22) (16, 22) (25, 22) (37, 22) (55, 22) (1, 37) (16, 37) (37, 37) (55, 37) |
formed by stations 9 and 7. Intersections 7, 10, and 12 are used as transfer points for the tandem layout. The detailed results of the variable path tandem layout for example 2 are presented in Table 8.

**Comparison of Variable Path Tandem Layout with Conventional Layout for Example 2**

As seen from the analysis, the tandem layout requires four zones. Therefore, four vehicles are required to operate the system. The average utilization per vehicle is equal to \((82.92\% + 99.28\% + 97.65\% + 34.00\%) / 4 = 78.46\%\). Again, this measure is meaningful only within the design requirement that workload feasibility per vehicle must be less than or equal to \(C\), where \(C \leq 100\%\).

For the conventional layout, given the system parameters (all parameters are the same as in the tandem layout) and the method of calculating vehicle requirement as given in Eqs. (6) to (12), the number of vehicles required is seven. The average vehicle utilization is 98.06%.

The utilization of 98.06% is significantly higher than the 78.46% obtained under the tandem layout. In addition, the conventional layout requires three more vehicles beyond that required by the tandem layout. The total vehicle use time, 188.31 min., for a tandem layout is less than half that of the vehicle use time, 411.86 min., for a conventional layout. Again, based on fleet size requirement and mean vehicle utilization, the result suggests that the tandem layout is superior to the conventional layout for the case presented. The difference in vehicle requirement between the two designs also presents opportunities for cost savings in moving from a conventional layout to a tandem layout. First, if the conventional system does not already exist but at the design model stage, then implementing a tandem layout instead of a conventional layout will result in savings of three vehicles (i.e., three vehicles fewer will be purchased). However, if the conventional system is already implemented, then a conversion to tandem layout will result in three excess vehicles. Depending on the specific situation, all or some of the excess vehicles can be resold or used as standby equipment to protect against vehicle breakdown. Reselling some of the excess capacity will reduce system cost.

**Balance of Vehicle Utilization Between Zones**

As was reported in the previous section, under the original design objective with a maximum allowable vehicle utilization of 100%, four vehicles were required by the tandem system described in example 2. The utilization of vehicles in the four zones were 82.92%, 99.28%, 97.65%, and 34.00%, respectively. The vehicle in the fourth zone is underutilized compared to those assigned to zones 1, 2, and 3. The utilization of the vehicles is unbalanced. In practice, this kind of imbalance is unacceptable and therefore would need to be corrected to bring the transport workload to balance between the vehicles or zones. Transport workload balance will necessitate a change in workstation makeup of the zones. This can be done by constraining the maximum allowable vehicle utilization to a desired value and then reinvoking the zone partitioning algorithm using the

<table>
<thead>
<tr>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stations</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>10, 11, 12, 14, 15, 18</td>
<td>8, 13, 16, 17, 19, 20</td>
</tr>
<tr>
<td>Transfer Points</td>
<td>10</td>
<td>10, 12</td>
<td>7, 12</td>
</tr>
<tr>
<td>Loaded Distance</td>
<td>339.00</td>
<td>306.00</td>
<td>360.00</td>
</tr>
<tr>
<td>Empty Distance</td>
<td>308.18</td>
<td>452.60</td>
<td>392.79</td>
</tr>
<tr>
<td>Number of Pickup</td>
<td>16.5</td>
<td>22.5</td>
<td>21.0</td>
</tr>
<tr>
<td>Number of Drop-off</td>
<td>16.5</td>
<td>22.5</td>
<td>21.0</td>
</tr>
<tr>
<td>Total Operation Time of AGV</td>
<td>49.75 min.</td>
<td>59.57 min.</td>
<td>58.59 min.</td>
</tr>
<tr>
<td>Vehicle Utilization</td>
<td>82.92%</td>
<td>99.28%</td>
<td>97.65%</td>
</tr>
</tbody>
</table>
constrained upper limit on vehicle utilization. This can be done iteratively by searching over the value of \( \mu \), the maximum allowable vehicle utilization, until a desired workload distribution is achieved.

Suppose the zones are designed using the upper bound of 0.9 (i.e., 90%) for maximum allowable vehicle utilization. Figure 6 shows the new tandem layout for example 2 when the maximum allowable vehicle utilization is constrained at 0.9. As it can be seen in Figure 6, four vehicles are still required at \( \mu = 0.9 \) for the tandem layout.

In this new tandem layout, vehicle 1 serves stations 1, 2, 3, 4, 5, and 6 in zone 1 and the vehicle utilization is 82.92% (49.75 min.). Vehicle 2 serves stations 10, 11, 14, 15, and 18 in zone 2 and the vehicle utilization is 75.38% (45.23 min.). Vehicle 3 serves stations 12, 16, 19, and 20 in zone 3 and the vehicle utilization is 60.73% (36.44 min.). The vehicle in zone 4 then serves stations 7, 8, 9, 13, and 17 and its utilization is 75.92% (45.55 min.). Intersections 10, 11, and 12 are used as transfer points. As it can be seen, the balance in the utilization of the vehicles among the zones has improved, although the same number of vehicles is used.

If the new workload distribution among the vehicles is acceptable, the search is terminated. Otherwise, a new value of \( \mu \) is specified and the partitioning algorithm is reinvoked. The process is continued until a desired workload balance is achieved or the balance can no longer be improved. Thereafter, the search is terminated.

**Conclusions**

In this research, the concept of a variable path tandem AGV system is presented. Similar to a tandem loop layout, in a variable path tandem layout only one dedicated vehicle serves all stations contained in a zone. A vehicle in a zone is also responsible for handling all unit loads in transit that pass through the zone. A variable path tandem system differs from a loop type system in the sense that there is routing flexibility in a zone or subdivision of the layout. The aisles in a zone do not necessarily form a loop but a mini-network. This way, a vehicle can travel along the shortest path between two points. This feature gives variable path tandem systems superiority over loop-type tandem systems. Furthermore, a variable path system also offers more layout flexibility because all pickup and drop-off points in a zone do not need to lie in a loop.

In this research, a heuristic algorithm for partitioning a conventional layout into a variable path tandem AGV system instead of a loop-based tandem AGV system is developed. The partitioning algorithm enables the grouping of workstations into single-vehicle zones or mini-networks while minimizing the number of zones and satisfying the total transport workload. Transfer points are simultaneously decided on during the network partitioning process. The algorithm assumes the existence of a conventional and unidirectional layout or a network model of such a layout. Because the problem of interest is one of converting an existing conventional layout or its equivalent model to a tandem layout, it is possible to make a direct comparison between the conventional and tandem AGV system with respect to system performance under the same operating conditions.

Results obtained from example problems show that variable path tandem AGV systems are very competitive with conventional AGV systems. In the two cases tested, the number of vehicles required of the tandem layout is fewer than that of the conventional layout. The vehicle use time for the tandem layout is also shorter than that of the conventional layout. Additionally, the control problem of the tandem layout is less complex than that of a conventional layout because only one vehicle is allowed to operate within a zone. No traffic congestion or conflict occurs in the tandem layout. Vehicles can travel bidirectionally or along the shortest path because only one vehicle operates in a zone. In a variable path tandem layout, all network links or aisles automatically convert to bidirectional aisles because no interference
between vehicles is expected. The routing flexibility provided by a bidirectional network of the tandem system over the directed network of a conventional system offers a competitive advantage.

References

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